POINT SETS AND CREMONA GROUPS. PART III

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In Part I[1] of this series projectively distinct sets P_n^k of n points in S_k were mapped upon points of a space $\Sigma_{k(n-k-2)}$ and a certain Cremona group $G_{n!}$ in Σ was obtained by permutation of the points of the set. In Part II[2] the G_n appeared as merely a subgroup of a more important group $G_{n,\,k}$ in $\Sigma_{\,k(n-k-2)}$ which also is defined by P_n^k. In particular the G_6, in Σ_4 attached to P_6^2 is a subgroup of the $G_{6,\,2}$ in Σ_4 which has the order 51840 and is isomorphic with the group of the lines on a cubic surface.

The purpose of this Part III is to utilize the $G_{6,\,2}$ in the problem of determining the lines of a cubic surface C^3. It appears that there is a one-to-one correspondence between the invariants of C^3 and the invariant spreads of $G_{6,\,2}$ in Σ_4. The lines of C^3 can be rationally expressed in terms of a solution of the form problem of $G_{6,\,2}$ by means of a typical representation of C^3 in the hexahedral form with the aid of the linear covariants of C^3. In order to solve the form problem of $G_{6,\,2}$ the simplest linear system of irrational invariants of C^3 is employed. This system is of dimension 9 and the members appear in Σ_4 as quintic spreads. Under the invariant subgroup $\Gamma_{6,\,2}$ of $G_{6,\,2}$ of index two this linear system separates into two skew linear systems each of dimension 4 with the important property that the members of the two systems are permuted under the operations of $G_{6,\,2}$ precisely as the points and S_3's of a linear space S_4 are permuted under the elements of a correlation group in S_4 whose collineation subgroup is the Burkhardt group G_{25920} in S_4. The form problem of $G_{6,\,2}$ can then be solved in terms of a solution of the form problem of G_{25920} by using the point invariants of G_{25920} and in addition five invariants of G_{25920} linear in the S_3 coördinates and of degrees 1, 7, 9, 13, 15 in the point coördinates.

The method for solving the form problem of G_{25920} is suggested by the properties of the normal hyperelliptic surface M_2^{18} of grade 3 in S_8 obtained parametrically by using 9 linearly independent theta functions of the third order and zero characteristic. The M_2^{18} admits a collineation group $G_{2,81}$ which contains 81 involutions. If I is one of these involutions the fixed S_3 and fixed S_4 of I meet M_2^{18} in 6 and 10 points respectively. The M_2^{18} is projected from the fixed S_4 upon the fixed S_3 into a doubly covered Weddle surface and from the fixed S_3 upon the fixed S_4 into a doubly covered 2-way N_2^6 which has a node

at α. If N_2^6 be projected from α it becomes a Kummer surface. There is a family of ∞^3 surfaces M_2^{18} with the same $G_{2\cdot 81}$. By projection we obtain a family of N_2^6's whose node α runs over a quartic spread J_4—the simplest invariant of G_{25920}. The 10 points in the S_4 of I run over the Hessian J_{10} of J_4. The spread J_4 is its own Steinerian and the polar cubic of a point α on J_4 as to J_4 is a Segre cubic spread with nodes at the 10 points on J_{10}, and of course a simple point at α. The point α on its polar cubic determines a binary sextic—the fundamental sextic of the hyperelliptic functions. In this way the solution of the form problem of G_{25920} in terms of hyperelliptic modular functions becomes apparent at once *in the special case* when $J_4 = 0$. This restriction is removed later by a conventional method. The conclusions above all are drawn from the existence of a set of 9 quadrics whose complete intersection is the normal spread M_2^{18} and whose coefficients are the modular forms α.

The above determination of the lines of C^3 differs from that of Klein[3] in that no equation of degree 27 or other resolvent equation is employed. All the processes are effected within the domain of the invariants and linear covariants of C^3. Klein also uses as fundamental form problem that of the Maschke collineation group in S_3 rather than the Burkhardt form problem. This implies the isolation of a root of the underlying binary sextic. The accessory irrationalities required are thereby somewhat simpler.

[1] These PROCEEDINGS, **1**, 245 (1915); *Trans. Amer. Math. Soc.*, **16**, 155 (1915). This series of investigations has been pursued under the auspices of the Carnegie Institution of Washington, D. C.

[2] These PROCEEDINGS, **2**, 244 (1916); *Trans. Amer. Math. Soc.*, **17**, 345 (1916).

[3] That an equation of degree 27 for the lines of a cubic surface could be solved by hyperelliptic modular functions was first pointed out by Klein, *J. Math., Paris*, Ser. 4, **4**, 169 (1888). His suggestions were elaborated by Witting, *Math. Ann., Leipzig*, **29**, 167 (1887); by Maschke, *Ibid.*, **33**, 317 (1889); and by Burkhardt, *Ibid.*, **35**, 198 (1890), **38**, 161 (1891), **41**, 313 (1893).

THE INTERFERENCES OF SPECTRA BOTH REVERSED AND INVERTED

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This is an interesting combination of the two methods of investigation hitherto given (Carnegie Publications, No. 249, 1916, §4) and not very difficult to produce. Retaining the adjustment for inverted spectra